Einstein Podolsky Rosen Paradoxon

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Why Einstein Podolsky Rosen Paradoxon?

because....

EPR-Source and Quantum Information Theory

What is a Paradoxon?

The word origin is old greek: $\underbrace{\pi\alpha\rho\alpha}_{contra}\underbrace{\delta o\xi o\nu}_{opinion}$

Definition

An apparently true statement that seems to lead to a contradiction or to circumstances that defy intuition

- Understand what Einstein Podolsky and Rosen are really saying

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- Find out if there is really a paradoxon

What is important for a physical Theory?

Distinguish between:

objective reality $\longleftrightarrow$ physical reality

What is Reality?

Definition

If, without in any way disturbing a system, we can predict with certainty (i.e with propability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.

When is a Theory Complete?

Definition

Every Element of the physical reality must have a counterpart in physical theory.

A Quantum System

Consider a quantum mechanical Discription of a particle with one degree of freedom:

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I.e. it exists an element of reality corresponding to A.

One particle system

we choose ψ:

$$\psi = e^{\frac{i}{\hbar} p_o x} \tag{2}$$

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and this means that the momentum p_o of the particle is real.

One dimensional Wavefunction

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for example if

$$q\psi = x\psi \neq a\psi \quad (5)$$

(where q is the coordinate operator)

In accordance with quantum mechanics one can speak about the probability of finding a value between a and b.

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In general this is true for all Operators A, B where the commutator is non zero, $[A, B] \neq 0$

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..that either

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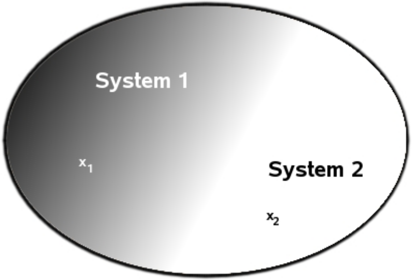
(1) the quantum mechanical description of objective reality via wavefunction is not **complete** or...

(2) if two operators do not commute, the physical quantities are not **simultanously real**.

Proof on the basis of a bipartite System



The Wave Function

$$\psi(x_1, x_2) = \sum_{n=1}^{\infty} \psi_n(x_2) u_n(x_1) \qquad (6)$$

where the $u_i(x_1)$ are Eigenfunctions of the Operator A

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The Wave Function

$$\psi(x_1, x_2) = \sum_{n=1}^{\infty} \varphi_n(x_2) v_n(x_1) \tag{8}$$

where the $v_i(x_1)$ are Eigenfunctions of the Operator B

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If we measure the quantity B and obtain b_r, we know the state of the system:

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If we measure the quantity B and obtain b_r, we know the state of the system:

$$\psi(x_1, x_2) = \varphi_r(x_2) v_r(x_1) \tag{9}$$

φ and ψ share the same reality!

As a consequence of to different meseaurements it is possible to attach the system 2, **two different** wavefunctions!!

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So it is possible to attach 2 wavefunctions to one reality!

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It can happen that this wavefunction obey noncommuting Operators!

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That this can happen is shown next.

Two particle System

Lets take the wavefunction for the system

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and $\psi_p(x_1) = \int_{-\infty}^{\infty} e^{-\frac{i}{\hbar}(x_2+x_o)p} dp$ is an Eigenfunction of $P_2 = -i\hbar\frac{\partial}{\partial x_2}$ with eigenvalue -p.

Two particle System

On the other hand we choose for B the coordinate of the particle in system one, $B = Q_1$ with the Eingenfunction

$$v_x(x_1) = \delta(x_1 - x_2) \tag{11}$$

Then it follows for the wavefunction of the system

$$\psi(x_1, x_2) = \int_{-\infty}^{\infty} \varphi_x(x_2) v_x(x_1) dx \tag{12}$$

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therefor φ_x has to be:

$$\varphi_x(x_2) = \int_{-\infty}^{\infty} e^{\frac{i}{\hbar}(x - x_2 + x_o)p} dp \overset{fourier \ repr. \ of \ delta}{\overset{\downarrow}{=}} 2\pi\hbar\delta(x - x_2 + x_o) \tag{13}$$

Two particle System

But this wavefunction is Eingenfunction of Q_2:

$$Q_2\varphi_x = 2\pi\hbar(x + x_o)\varphi_x \tag{14}$$

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It is essential that $[P_2, Q_2] = i\hbar$!

Summary

System 1	System 2
$u_p(x_1) \Leftrightarrow A$	$\psi_p(x_2) \Leftrightarrow P_2$
$v_x(x_1) \Leftrightarrow A$	$\varphi_p(x_2) \Leftrightarrow Q_2$
	$\Rightarrow \varphi$, ψ are wavefunctions of noncommuting operators!

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	$\Rightarrow \varphi$, ψ are wavefunctions of noncommuting operators!

So **it is possible** to find wavefunction, which represents the same reality, who are eigenfunctions of noncommuting operators!

What does this mean?

It means that by measuring the Observables A and B in System 1 it is possible to say wiht certainty the physical quantities of the P_w and Q_2.

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So the completeness of the quantum mechanical description via wavefunction implies that two noncommuting operators have simultanously reality.

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So the completeness of the quantum mechanical description via wavefunction implies that two noncommuting operators have simultanously reality.

This leads to the negation of (2) and we are left with the only other consequence:
(1) the quantum mechanical description of objective reality via wavefunction is not **complete**

Statistical Interpretation

Even if quantum mechanics is not complete - or how we would say today: quantum mechanic is nondeterministic - it is possible to work with the concept of wavefunction, but in a statistical way:

$$\Psi = \sum_n c_n \psi_n \tag{15}$$

where c_n are totally uknown coefficients.

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- Einstein Podolsky Rosen have proofen that quantum mechanics is not comlete
- What is the connection to Quantum Information Thery?
- Where is the paradox?

Literature

A. Einstein, B.Podolsky and N. Rosen *Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?*. Physical Review Letters, 1935 , pp 777-780